A small tour of optimization models

Theory of games and markets with examples

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OPTIMIZATION AND APPLICATIONS

Consulting:
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Web-sites:

http://soften.ktu.lt/~mockus

http://pilis.if.ktu.lt/~jmockus

http://eta.ktl.mii.lt/ mockus

http://mockus.us/optimum (short)

Optimality

Objective function

$$f(x),\ x = (x_1, ..., x_m). \tag{1}$$

Global minimum

$$f(x_A) \leq f(x)\ x \in A, \tag{2}$$

or

$$x_A = \arg\min_A f(x). \tag{3}$$

Here A is a feasible region.

Local minimum

$$f(x_\epsilon) \leq f(x)\ x \in \epsilon. \tag{4}$$

Here ϵ is a vicinity of x_ϵ.

Discrete and convex optimization

Discrete optimization

if variables x_i are discrete.

Linear programming

if

$$f(x) = \sum_{i=1}^{m} c_i x_i, \tag{5}$$

$$A: \sum_{i=1}^{m} a_{ij} x_i \geq b_j, \; j = 1, ..., m, \; x_i \geq \; 0. \tag{6}$$

Convex programming

if both objective $f(x)$ and feasible region A are convex.

Diet problem

$$min_x \sum_{i=1}^{m}(c_i - s_i)x_i + g(\sum_{i=1}^{m} a_{i1}x_i - b_1), \quad (7)$$

$$\sum_{i=1}^{m} a_{ij}x_i \geq b_j,\ j = 1, ..., n \quad (8)$$

$$x_i \geq 0,\ i = 1, ..., m \quad (9)$$

For example,
c_1 is a price of bread,
a_{11} are calories of bread,
b_1 are necessary calories.
s_i is the taste (expressed in money units) , g beauty (expressed in money units)
Solving by standard simplex or interior point methods.

Diverse diet problem

$$min_x \sum_{i=1}^{m}(c_i - s_i)x_i + g(\sum_{i=1}^{m} a_{i1}x_i - b_1) - d(\sum_{i=1}^{m} d_i x_i), \qquad (10)$$

$$\sum_{i=1}^{m} a_{ij}x_i \geq b_j, \ j = 1, ..., n \qquad (11)$$

Here
d is the taste of food diversity (expressed in money units),
d_i is the dish indicator: $d_i = 1$, if x_i is a dish, $d_i = 0$, otherwise,
If x_i is a dish then $x_i = int$ and $0 \leq x_i \leq 1$.

Longer term diet problem

$$min_x \sum_{i=1}^{m}(c_i - s_i)x_i + g(\sum_{i=1}^{m} a_{i1}x_i - Tb_1) - d(\sum_{i=1}^{m} d_i x_i), \qquad (12)$$

$$\sum_{i=1}^{m} a_{ij}x_i \geq Tb_j, \; j = 1, ..., n \qquad (13)$$

$$\sum_{i=1}^{m} d_i x_i \leq T, \; \sum_{i=1}^{m} d_i \geq T. \qquad (14)$$

Here d is the taste of food diversity, T is the time period, d_i is the dish indicator: $d_i = 1$, if x_i is a dish, othervise $d_i = 0$. If x_i is a dish then $x_i = int$ and $0 \leq y_i \leq 1$.

Combinatorial diet problem

$$min_x \sum_{i=1}^{m} (c_i - s_i)x_i + g(\sum_{i=1}^{m} a_{i1}x_i - Tb_1) - d(x), \tag{15}$$

$$\sum_{i=1}^{m} a_{ij}x_i \geq Tb_j, \ j = 1, ..., n \tag{16}$$

Here $x = (x_i, i = 1, ..., m)$, T is the time period, x_i is a number of dishes (meals) i during time T, thus $x_i = 1, 2, ..., T$, $d(x)$ is a function defining the taste of food diversity (for example, $d(x)$ could be a number of different non-zero components x_i in the diet x),
Bayesian Heuristic Approach (BHA) is recommended method for improving a user defined initial diet by optimizing parameters of Simulated Annealing (SA).

Solving combinatorial diet problem

Step 1. Set an initial diet $x = x^1$ and evaluate its quality by calculating the sum:

$$D(x^1) = \sum_{i=1}^{m} (c_i - s_i) x_i^1 + g(\sum_{i=1}^{m} a_{i1} x_i^1 - T b_1) - d(x^1), \qquad (17)$$

Step 2. Generate next diet x^2 by changing randomly some dishes and evaluate the quality $D(x^2)$.
Step 3. If $h_2 = D(x^2) - D(x^1) \leq 0$ go to x^2.
If $h_2 > 0$ go to x^2 with probability r_2;
keep x^1 and return to step 1 with probability $1 - r_2$.
Probability r_2 is generated by SA formula **??** defined in the slide "Simulated Annealing"
Parameter x of SA is optimized using GMJ system.

Defining SA parameters

Step 1. Generate probability r_2 by SA formula 173.
Step 2. Optimize parameter x of SA by GMJ.
The standart reference to GMJ is this:

```
public Domain domain ()
return domain;
public double f (Point pt)
..............
return f
```

Here 'domain' defines constraints of SA variables,
'pt' is vector of SA variables,
'f' is the 'goodness' function of the best diet
after 'IT' iterations using fixed SA variables.

Simplex algorithm

Simple example:

$$min_x z \tag{18}$$

$$z = x_1 - x_3 \tag{19}$$

$$x_1 + x_2 + x_3 = 1, x_i \geq 0 \tag{20}$$

Here x_2 base variables,
$x_1,\ x_3$ free variables. $x_1 = 1$ base solution obtained when free variables are equal to zero
$x_1 = x_3 = 0$, then $z = 0$.
This base solution is improved by increasing x_3, because $c_3 = -1 < 0$.
New base solution $x_3 = 1, x_1 = x_2 = 0$, where $z = -1$, can't be improved,
since both free variables are non-negative $c_1 = 1, c_2 = 0$.